Logic PhD Exam, August 2020.

Solve 5 problems of the following; at least one from each section.

A. Set Theory.

1. State Martin's Axiom for $\aleph_1$ and prove that it implies the negation of the Continuum Hypothesis.
2. State and prove Fodor's pressing-down theorem.
3. Let κ be an uncountable cardinal. Show that there is an ultrafilter on κ containing no sets of cardinality smaller than κ.

B. Computability.

1. State two different common definitions of the class of computable functions and sketch the proof that they give the same class.
2. Define the many-one reducibility and sketch the proof that there is a many-one degree strictly between $\mathbf{0}$ and $\mathbf{0}'$.
3. Give an example of an infinite decidable structure and infinite undecidable structure and prove that they are such.

C. Model theory.

1. What is quantifier elimination for theories? Give an example of a theory which has it, and prove that it has it.
2. State the Los' theorem and prove it.
3. State and prove the Tarski criterion for elementarity of submodels.