WORK EACH PROBLEM ON A SEPARATE PAGE. PUT YOUR NAME ON EACH PAGE.

1. Let $\lambda > 1$. How many solutions does the equation $\lambda - z - e^{-z} = 0$ have in the right half plane $\{z : \text{Re}z > 0\}$.

2. Determine all one-one entire functions.

3. Determine the order of the entire function $\cosh(\sqrt{z})$.

4. Does there exist a function analytic in $|z| < 1$ satisfying $|f| < 1$, $f(0) = 0$ and $f(1/2) = 3/4$?

5. Let $a_n \geq 0$ and $f = \sum_{n=0}^{\infty} a_n z^n$ with radius of convergence $\infty > R > 0$. Show that $z = R$ is a singularity of f. (Suggestion: Show, if $R > r > 0$ and

$$\sum \frac{f^{(n)}(r)}{n!}(z-r)^n$$

has radius of convergence R', then for each $|z_0| = r$

$$\sum \frac{f^{(n)}(z_0)}{n!}(z-z_0)^n$$

has radius of convergence at least R'. Conclude f is analytic in $|z| < R' + r$.)

6. Show that $f(z) = \sum \frac{z^n}{n^2}$ is analytic at each point $|z| = 1$, except $z = 1$.

7. Suppose f is entire of order ρ. Let $n(r)$ denote the number of zeros of f in the disc $|z| < r$. Show

$$\limsup_{r \to \infty} \frac{\log(n(r))}{\log(r)} \leq \rho.$$

8. Evaluate the integral

$$\int_0^{\infty} \frac{t^a}{1+t+t^2}dt \quad -1 < a < 1.$$

(Suggestion: Integrate along the keyhole contour, cut along the nonnegative real axis consisting of the two edges of the real axis from ϵ to R and a small circle of radius ϵ and a large circle of radius R about the origin.)

9. Let A denote the region inside the circle $|z-2| = 2$ and outside the circle $|z-1| = 1$. Construct a conformal map of A onto the unit disc. (Suggestion: Where does the map $1/z$ carry A?).