

Logic PhD Exam, May 2026.

Solve 5 problems of the following; at least one from each section.

A. Set Theory.

1. State and prove the Δ -system lemma.
2. Sketch a proof that the comprehension axiom schema holds in the constructible universe.
3. Given a compact Polish space X and a Polish space Y , explain how the space $C(X, Y)$ of all continuous functions is topologized. Identify a compatible metric on it and prove that it is complete.

B. Computability.

1. State Gödel's diagonalization lemma and provide a proof.
2. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(0) = 0$, $f(1) = 1$, and $f(n+2) = 2f(n) + 3f(n+1)$ is recursive.
3. Sketch a proof that the structure $\langle \mathbb{N}, 0, 1, \leq, +, \cdot \rangle$ is undecidable.

C. Model theory.

1. What is a Fraïssé class? State the Fraïssé theorem regarding the existence of a limit model for such a class.
2. What is quantifier elimination? Provide an example of a theory which has infinite models and quantifier elimination. Sketch a proof that the theory has quantifier elimination.
3. Find a complete theory with more than one countable model up to isomorphism. Provide two nonisomorphic countable models of the theory. Explain why they are nonisomorphic.