1st Semester Topology Exam
August, 2020

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Is it true that every continuous map $f : [0, 1] \to (0, 1)$ has a fixed point. Justify your answer.
2. Show that every connected normal space having more than one point is uncountable.
3. Show that the set of algebraic numbers is countable (A number $a \in \mathbb{R}$ is called *algebraic* if a is a root of a polynomial with rational coefficients).
4. Show that Greek letter theta Θ is not homeomorphic to the Mercedes symbol S.
(Formally, $\Theta = S^1 \cup [-1, 1] \times \{0\}$ and $S = S^1 \cup I_1 \cup I_2 \cup I_3$ are subspaces of $\mathbb{R}^2$ where S^1 is the unit circle and I_k, $k = 1, 2, 3$, are the closed intervals in $\mathbb{R}^2$ joining the origin with the points $(0, 1)$, $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$, and $(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$.)
5. Let A be a proper subset of X and let B be a proper subset of Y. If X and Y are connected, show that $(X \times Y) - (A \times B)$ is connected.

Answer the following with complete definitions or statements or proofs.

6. State the Extreme Value Theorem.
7. State the Tietze Extension Theorem.
8. Is the space $I^{\mathbb{N}}$ separable? What about I^I? Here $I = [0, 1]$ and $\mathbb{N}$ is the set of natural numbers.
9. Prove that the space $X = \{(x, y) \mid xy = 1\} \subset \mathbb{R}^2$ is locally compact? What is its one-point compactification?
10. Is every path connected space locally connected?