Lecture 32: (Sec. 6.3)

Area and the Definite Integral; The Fundamental Theorem of Calculus

How to find the area **under the graph of** f on the interval $[a, b]$ or from a to b?

ex. Let $f(x) = \sqrt{x+1}$ and consider the area under the graph of f on $[0, 2]$.

We approximate the area of the region using four rectangles:

1) Divide the interval $[0, 2]$ into four subintervals of equal length.

2) Find the midpoint of each of the subintervals.

3) Construct four rectangles with the subintervals as the base and the height, the function value of the midpoint of each interval:

4) Approximate the area of the region.

Note: The area is approximately 2.79975.

What if we use eight rectangles?

Area ≈ 2.7979836

Later we can compute the area more precisely as 2.79743.

We can generalize these examples to define area under the graph of f on $[a, b]$, where f is continuous and nonnegative on that interval.

The sum above is called

Area under the graph of a function

Def. Let f be a nonnegative continuous function on $[a, b]$. Then the area of the region under the graph of f on $[a, b]$ is

where $\Delta x =$

and $x_1, x_2, ..., x_n$ are

NOTE:

We can extend this idea to a function which may not be nonnegative:

<u>**Def.**</u> Let f be defined on $[a, b]$. The **definite integral of f on** $[a, b]$ is

$$\int_a^b f(x)\,dx =$$

where $\Delta x =$

and $x_1, x_2, ..., x_n$ are

a is

b is

<u>**Def.**</u> f is **integrable** on $[a, b]$

Theorem: Let f be continuous on $[a, b]$. Then f is

Consider the relationship between the definite integral and area:

If f is continuous and nonnegative on $[a, b]$,

$$\int_a^b f(x)\, dx \text{ is}$$

ex. Sketch the region corresponding to the following definite integral; then evaluate using geometric formulas.

$$\int_0^2 (4 - x)\, dx$$

What if $f(x)$ is negative for some x on $[a, b]$?