1st Year 2nd Semester Topology Exam
August, 2021

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper. Each problem is worth 15 points.

1. Show that S^2 is not homeomorphic to S^4.
2. Show that the Stone-Čech compactification βX of X is connected if and only if X is connected.
3. Let $X_{\geq 0}$, $Y_{\geq 0}$, and $Z_{\geq 0}$ denote the nonnegative parts of the x-axis, the y-axis, and the z-axis in $\mathbb{R}^3$ respectively. Let $A = \mathbb{R}^3 \setminus (X_{\geq 0} \cup Y_{\geq 0} \cup Z_{\geq 0})$. Is the fundamental group $\pi_1(A)$ abelian?
4. Show that the torus T is not homeomorphic to the Klein bottle K.
5. By the definition the triod (or the letter T space) is a space homeomorphic to $X = ([0, 1] \times \{0\}) \cup (\{0\} \times [-1, 1]) \subset \mathbb{R}^2$. Show that every continuous map of the triod to itself has a fixed point.

Answer the following with complete definitions or statements or proofs. Each problem is worth 5 points.

6. Does the sequence $f_n : \mathbb{R} \to \mathbb{R}$, $f_n(x) = \ln(x^{\frac{1}{n}})$, converge in
(a) the point-wise convergence topology?
(b) the uniform topology?
(c) compact open topology?
7. State the Borsuk-Ulam Theorem for S^2.
8. Let $X = \mathbb{R}$ and $A = \mathbb{Z}$ be the reals and the integers respectively. Is the quotient space $Y = X/A$ metrizable?
9. State the Seifert-van Kampen Theorem.
10. State the Urysohn Lemma.