1st Year 2nd Semester Topology Exam
May, 2020

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Show that S^2 is not homeomorphic to S^3.

2. Prove that the set of irrational numbers $\mathcal{I} \subset \mathbb{R}$ cannot be presented as a countable union of closed sets each of which has empty interior in $\mathcal{I}$.

3. Show that for every continuous map $f : S^1 \to S^1$ there is $n \in \mathbb{Z}$ such that f is homotopic to $z^n : S^1 \to S^1$.

4. Show that every map $f : S^2 \to T$ of the 2-sphere to the torus T is null-homotopic.

5. Show that the torus T is not homeomorphic to the surface of genus 2, $M_2 = T \# T$.

Answer the following with complete definitions or statements or proofs.

6. Does the sequence $f_n : \mathbb{R} \to \mathbb{R}$, $f_n(x) = \sqrt{x}/n$, converge in
(a) the point-wise convergence topology?
(b) the uniform topology?
(c) compact open topology?

7. State the Borsuk-Ulam Theorem for S^2.

8. Is the space I^I separable where $I = [0, 1]$?

9. State the Seifert-van Kampen Theorem.

10. State and prove the Tychonoff Theorem.