1st Semester Topology Exam

May 2023

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper (10 pts each problem).

1. Let X be a locally connected space. Show that each component of X is closed.

2. Is $\mathbb{R}_\ell$ a Baire space?

Here $\mathbb{R}_\ell$ denotes the reals with the lower limit topology, i.e. the topology defined by the basis $\{[a,b) \mid a,b \in \mathbb{R}\}$.

3. Let $X = \mathbb{Z} \times [0,1] \subset \mathbb{R}^2$. Let $Y = \mathbb{Z} \times (0,1]$ and $A = \mathbb{Z} \times \{0\}$ be subspaces of X. Is the one point compactification of Y homeomorphic to the quotient space X/A?

4. Show that the ordered square I_O^2 is not path connected.

Here I_O is the square $[0,1] \times [0,1]$ given the lexicographic order topology.

5. Show that the power set $\mathcal{P}(X)$ has strictly greater cardinality than X.

Answer the following with complete definitions or statements or short proofs (5 pts each problem).

6. Show that the circle S^1 is not homeomorphic to the 2-sphere S^2.

7. Show that the 2-sphere S^2 is not homeomorphic to the plane $\mathbb{R}^2$.

8. Is $\mathbb{R}_\ell$ regular?

9. State the Baire Category Theorem

10. Show that a connected metric space cannot be infinite countable.

11. State the Intermediate Value Theorem

12. Give definition of a quotient map.

13. Are the spaces $\mathbb{R} \times [0,\infty)$ and $[0,\infty) \times [0,\infty)$ homeomorphic ?

14. Is every compact Hausdorff space completely regular ?

15. Is every continuous injective map $f : \mathbb{R} \to \mathbb{R}^2$ an embedding?