# 1st Year Topology Exam. Part 1

January, 2023

**Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper**.

1. Show that a connected metric space having more than one point is uncountable.
2. Give definition of retraction. Show that a retraction is a quotient map.
3. Is the set of rationals $\mathbb{Q}$ in the subspace topology locally compact?
4. Show that the set of irrational numbers in $\mathbb{R}$ is a Baire space.
5. Let $f : X \to X$ be a map of a compact metric space $(X, d)$ to itself that satisfies the following condition $d(f(x), f(y)) < d(x, y)$ for all $x \neq y$. Prove that $f$ is continuous. Show that $f$ has a unique fixed point.

**Answer the following with complete definitions or statements or short proofs.**

6. State the Intermediate Value Theorem.
7. Show that any continuous map $f : S^2 \to (0, 1)$ of the 2-sphere $S^2$ to the interval $(0, 1)$ is neither surjective nor injective.
8. State the Cantor-Schroeder-Bernstein Theorem.
9. Is the space $\mathbb{R}_\ell$ connected? path connected?
10. What is a basis of a topology? Does the set

$$\{(a, b] \cup [c, d) \mid a < b < c < d\}$$

form a basis of a topology on $\mathbb{R}$?