1st Semester Topology Exam
August, 2021

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper. Each problem is worth 15 points.

1. Is it true that every continuous map $f : [0, 1] \to [0, 1)$ has a fixed point. Justify your answer.
2. Show that every compact Hausdorff space is regular.
3. Is it true that
(a) every continuous injective map $f : (0, 1) \to \mathbb{R}$ is an embedding?
(b) every continuous injective map $f : (0, 1) \to \mathbb{R}^2$ is an embedding?
4. Show that Greek letter theta Θ is not homeomorphic to the figure eight 8.
(Formally, $\Theta = S^1 \cup [-1, 1] \times \{0\}$ and $8 = S^1_- \cup S^1_+$ are subspaces of $\mathbb{R}^2$ where S^1_- is the unit circle centered at the point $(0, -1)$ and S^1_+ is the unit circle centerd at $(0, 1)$.
5. (a) Let $A \subset \mathbb{R}^2$ be a countable set. Prove that $\mathbb{R}^2 \setminus A$ is path connected.
(b) Let $B \subset \mathbb{R}^3$ be the union of a countable family of lines in $\mathbb{R}^3$. Prove that $\mathbb{R}^3 \setminus B$ is path connected.

Answer the following with complete definitions or statements or proofs. Each problem is worth 5 points.

6. State the Extreme Value Theorem.
7. State the Tietze Extension Theorem.
8. Let $\mathbb{R}_\ell$ denote the reals with the lower limit topology. Is the subspace $\mathbb{R}_\ell \cap [0, 1]$
(a) compact?
(b) connected?
9. Is every compact Hausdorff space normal?
10. Give definition of a locally connected space. Is every path connected space locally connected?