1st Semester Topology (MTG5316) Exam, May 2021

Part 1. Work each of the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper. These five problems will be worth 15 points each.

1. Let S be a set, and let $\mathscr{P}(S)$ denote the set of subsets of S. Prove that there is no surjective function $f : S \to \mathscr{P}(S)$.

2. Let X and Y be connected topological spaces. Prove that $X \times Y$ is connected, where $X \times Y$ has the product topology.

3. Prove that every compact, metrizable space has a countable basis.

4. Prove that the one-point compactification of $\mathbb{Z}_+$ is homeomorphic to the subspace $\{0\} \cup \{\frac{1}{n} : n \in \mathbb{Z}_+\}$ of $\mathbb{R}$.

5. Recall that $\mathbb{R}_K$ denotes the real line with the K-topology, and the set K is given by $K = \{\frac{1}{n} : n \in \mathbb{Z}_+\}$. Let Y denote the quotient space obtained from $\mathbb{R}_K$ by collapsing the set K to a point. Prove that Y satisfies the T_1 axiom, but is not Hausdorff.

Part 2. Answer the following with complete definitions or statements or short proofs. These five problems will be worth 5 points each.

6. In the finite complement topology on $\mathbb{R}$, to what point or points does the sequence $x_n = \frac{1}{n}$ converge?

7. Is every simply ordered set a Hausdorff space in the order topology?

8. Define compact, limit point compact, and sequentially compact.

9. If $f : X \to Y$ is a continuous surjective function, and X is path connected, must Y be path connected?

10. State the Intermediate Value Theorem.