Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. (10 points) State (3 points) and prove (7 points) Lagrange's Theorem.

2. Let G be a finite group, and let H and J be normal subgroups of G.
   (a) (5 points) Prove that $H \cap J$ is a normal subgroup of G;
   (b) (5 points) Prove that if $H \cap J = 1$ then, for every $h \in H$ and every $j \in J$, we have $hj = jh$.

3. (10 points) Let G be a group acting on a non empty set S, and H be a normal subgroup of G. Assume that for any $x_1, x_2 \in S$ there is a unique $h \in H$ such that $h(x_1) = x_2$. For $x \in S$ let G_x be the stabilizer of x. Show that for any $x \in S$, G is a semi-direct product of G_x and H. Do not assume that G is finite.

4. (a) (5 points) Prove that a group G of order 245 has a unique subgroup H of order 49.
   (b) (5 points) Show that if the above subgroup H is cyclic then G is abelian.

5. (10 points) Give the definition of a solvable (finite) group. (You may give any of the equivalent definitions.) Prove, using your definition, that the symmetric group on four letters is solvable.

6. (a) (5 points) Find all the conjugacy classes of elements of order 2 in S_7.
   (b) (5 points) Do the same for A_7. In both cases, justify your answer.