Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ________________________________

Problems to be graded: 1 2 3 4 5 6

1. Give an example of each, with a few words of explanation.
   (a) (2 points) A finite abelian simple group.
   (b) (2 points) A nonabelian simple group.
   (c) (3 points) A finite nilpotent group which is not a p-group.
   (d) (3 points) A finite group which is solvable but not nilpotent.

2. Consider the symmetric group S_6, and the corresponding alternating group A_6.
   (a) (5 points) How many elements of order 4 are there in S_6 ? Justify your answer.
   (b) (5 points) How many elements of order 4 are there in A_6 ? Justify your answer.

3. (10 points) Let G be a finite p-group and suppose that G acts on a finite set S. Denote by S^G set of all elements of S fixed by every element of G. Show that $|S| \equiv |S^G| \pmod{p}$, where $|A|$ denotes the cardinality of the set A.

4. Let G be a group of order $32 \cdot 11 \cdot 41$.
   (a) (3 points) Show that every 41-Sylow subgroup of G is normal.
   (b) (3 points) Show that if H is a group of order $32 \cdot 11$ then every 11-Sylow subgroup of H is normal.
   (c) (4 points) Show that G has a normal cyclic subgroup of order $451 = 11 \cdot 41$.

5. Let G be a group and let $g \in G$.
   (a) (2 points) Define what is meant by the order of g.
   (b) (4 points) Suppose that g has infinite order, and suppose that N is a finite normal subgroup of G. Prove, from your definition, that the element $gN \in G/N$ has infinite order.
   (c) (4 points) Suppose that g has finite order, suppose that N is a finite normal subgroup of G and, suppose that $|N|$ is relatively prime to the order of g. Prove, from your definition, that the element $gN \in G/N$ has the same order as the element $g \in G$.

6. (10 points) Show that for any $n \geq 5$, the alternating group A_n is the only nontrivial proper normal subgroup of the symmetric group S_n.