First Semester Algebra Exam – May 2020
Department of Mathematics, University of Florida

NAME:

Problems: 1 2 3 4 5 6

Instructions: Do four out of the six problems below. Indicate on the line above the problems you have done by circling the number. If you hand in more than four problems, only the first four will be graded. You may (within reason) use results from the text if you state them fully.

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1. Suppose that M and N are normal subgroups of a group G (i) Show that $G/(M \cap N)$ is isomorphic to a subgroup of $G/M \times G/N$. (ii) Show that if $G = MN$, then in fact $G/(M \cap N) \simeq G/M \times G/N$.

2. (i) Find a representative for each conjugacy class of elements of order 4 in A_8. (ii) Describe the centralizer of each such representative as a suitable semidirect product.

3. Suppose G is a group of order $351 = 3^3 \cdot 13$. Show that G has a normal Sylow subgroup.

4. List all isomorphism classes of abelian groups of order 16. One of these has an automorphism of order 5; which is it?

5. Suppose G is a solvable group, $H \subseteq G$ is a nontrivial normal subgroup. Show that H has a nontrivial abelian subgroup that is normal in G (consider the derived series of H).

6. Construct four nonisomorphic groups of order 28 (use a semidirect product for one case).