Topology Ph.D. Exam

August, 2017

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper (10 pts each).

1. Let X be a connected metric space having more than one point. Can X be countable?

2. Compute $\pi_1(T^2)$ and $H_*(T^2)$ where $T^2 = S^1 \times S^1$ is the 2-dimensional torus.

3. Does there exist a map of degree 2

(a) from S^2 to the torus T^2 ?

(b) from T^2 to S^2 ?

4. Show that every compact Hausdorff space is normal.

5. Does there exists a covering space of the 2-dimensional sphere with three points deleted with nontrivial abelian fundamental group?

Answer the following with complete definitions or statements or short proofs (5 pts each).

6. State the Baire Category Theorem.

7. State the homology Mayer-Vietoris Theorem.

8. State the Urysohn Lemma.

9. Do the following short exact sequences always split

$$0 \to A \to B \to \mathbb{Z}^2 \to 0$$

$$0 \to \mathbb{Z} \to A \to B \to 0 \ \ ?$$

10. Compute the Euler characteristic $\chi(\mathbb{R}P^2 \times S^2 \times S^2 \times S^3)$.

11. Give an example of $A \subset X$ such that A is a retract of X but not a deformation retract.

12. State the Lefschetz Fixed Point Theorem.

13. Can irrational numbers be presented as a countable union of closed in $\mathbb{R}$ subsets?

14. Draw a picture of the universal cover of the 2-sphere with the segment joining the north and south poles.

15. List all i for which there is a closed orientable 6-manifold M with $H_i(M) = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$.