

Topology Ph.D Exam

May 2016

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper

1. Show that there is no map $S^2 \rightarrow T^2$ of degree 2. Does there exist a map $f : T^2 \rightarrow S^2$ of degree 2?
2. Does every function $f(x) = x^2$ admit a continuous extension $\bar{f} : \beta\mathbb{R} \rightarrow \mathbb{R}$ to the Stone–Čech compactification?
3. Is the 2-sphere S^2 with k points removed homeomorphic to a topological group for
(a) $k = 0$? (b) $k = 1$? (c) $k = 2$? (d) $k = 3$?
4. Prove that the topologist sin curve is not path connected.
5. Suppose that $X = C_1 \cup C_2$ where each C_i is homeomorphic to the topologist sin curve. Can X be path connected?

Answer the following with complete definitions or statements or short proofs.

6. Why the n -sphere S^n is not homeomorphic to the n -space \mathbb{R}^n ?
7. State the Contraction Mapping Theorem.
8. Compute the Euler characteristic $\chi(\mathbb{C}P^5 \times \mathbb{R}P^5 \times S^5 \times S^2)$.
9. State the Baire Category Theorem.
10. Does a surface of genus 2 admit a nonzero vector field?
11. State the Five Lemma.
12. What can you say about the k -th cohomology group of a closed oriented n -manifold for
(a) $k = n$?
(b) $k = n - 1$?
13. Does there exist a covering space of the figure eight that has a non-trivial abelian fundamental group?
14. State the Kunneth Formula for homology.
15. State the Lefschetz Fixed Point Theorem.