Topology Ph.D. Exam
May, 2014

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Compute $\pi_1(T^2)$ and $H_*(T^2)$ where $T^2 = S^1 \times S^1$ is the 2-dimensional torus.
2. Let X be a connected completely regular topological space having more than one point. Can X be countable?
3. Prove that there is no map of degree 3 from S^2 to the torus T^2.
4. Does there exists a covering space of the 2-dimensional sphere with four points deleted with nontrivial abelian fundamental group?
5. Show that the 2-dimensional sphere with four points deleted cannot be a topological group.

Answer the following with complete definitions or statements or short proofs.

6. State the Baire Category Theorem.
7. State the homology Mayer-Vietoris Theorem.
8. State the Urysohn Lemma.
9. Do the following short exact sequences always split

$$0 \to A \to B \to \mathbb{Z}^2 \to 0$$

$$0 \to \mathbb{Z} \to A \to B \to 0 \ \ ?$$

10. Compute the Euler characteristic $\chi(\mathbb{R}P^2 \times S^1 \times S^2 \times S^3)$.
11. Give an example of $A \subset X$ such that A is a retract of X but not a deformation retract.
12. State the Lefschetz Fixed Point Theorem.
13. Can irrational numbers be presented as a countable union of closed in $\mathbb{R}$ subsets?
14. Draw a picture of the universal cover of the 2-sphere with the segment joining the north and south poles.
15. List all i for which there is a closed orientable 6-manifold M with $H_i(M) = \mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$.