Topology Ph.D. Exam
May 20, 2008

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Compute $\pi_1(S^2)$ and $H_*(S^2)$.
2. Let X be a connected completely regular topological space having more than one point. Can X be countable?
3. Does there exists a covering space of the figure eight with non-trivial abelian fundamental group?
4. Show that the 2-dimensional sphere with four points deleted cannot be a topological group.
5. Prove that there is no map of degree two from S^2 to the torus T^2.

Answer the following with complete definitions or statements or short proofs.

6. State the Baire Category Theorem.
7. State the homology Mayer-Vietoris Theorem.
8. State the Urysohn Lemma.
9. State the Five Lemma.
10. Compute the Euler characteristic $\chi(RP^4 \times S^3 \times T^2)$.
11. Define retraction and deformation retraction.
12. State the Lefschetz Fixed Point Theorem.
13. Can irrational numbers be presented as a countable union of closed in $\mathbb{R}$ subsets?
14. Draw a picture of the universal cover of the 2-sphere with the segment joining the north and south poles.
15. Describe all connected subsets of $\mathbb{R}$.