Topology Ph.D. Exam

September 2004

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper

1. Show that $\pi_1(S^2)$ is trivial.

2. Show that there is no retraction of the Mobius band onto its boundary.

3. Does there exists a covering space of the figure eight with nontrivial abelian fundamental group?

4. Show that the 2-dimensional sphere with three points deleted cannot be a topological group.

5. Let M be a 4-dimensional closed simply connected manifold. Show that every continuous map $f : M \to M$ which is homotopic to the identity has a fixed point.

Answer the following with complete definitions or statements or short proofs.

6. State the Baire Category Theorem.

7. State the Lefschetz Fixed Point Theorem.

8. State the Jordan Curve Theorem.

9. State the Five Lemma.

10. Show that no covering of the 2-torus has the homotopy type of a figure eight.

11. Let $S_r(x, y)$ denote a circle of radius r centered at $(x, y) \in \mathbf{R}^2$. Let ΣR be the suspension over Hawaiian ring $R = \cup_{n=1}^{\infty} S_{\frac{1}{n}}(\frac{1}{n}, 0)$. Is ΣR

(a) locally path connected?

(b) locally simply connected?

(c) semi-locally simply connected?

12. State the Tietze Extension Theorem.

13. Define retraction and deformation retraction.

14. State the Universal Coefficient formula for homology.

15. Let X be a complete metric space such that no point is isolated. Can X be countable?