

Topology Ph.D. Exam

January 2004

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper

1. Show that no covering of the 2-torus has the homotopy type of a figure eight.
2. Show that there is no retraction of the Mobius band onto its boundary.
3. Does there exist a covering space of the figure eight that has a non-trivial abelian fundamental group?
4. Show that the 2-torus with a point deleted does not admit a structure of a topological group.
5. Show that the space $\mathbb{R}^2 \setminus A$ is path connected for any countable set A .

Answer the following with complete definitions or statements or short proofs.

6. State the Seifert-van Kampen Theorem.
7. State the Lefschetz Fixed Point Theorem.
8. State the Baire Category Theorem.
9. State the Arzela-Ascoli Theorem.
10. Let X be a finite complex. Find $\chi(X \times S^n)$.
11. State the Five Lemma.
12. State the Poincare Duality Theorem.
13. Define retraction and deformation retraction.
14. State the Mayer-Vietoris exact sequence for cohomology.
15. Let X be a complete metric space such that no point is isolated. Can X be countable?