Topology Ph.D. Exam

September 2002

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper

1. Use the Mayer-Vietoris sequence for reduced singular homology to compute $H_*(S^n)$.

2. Show that there is no retraction of the Mobius band onto its boundary.

3. Does there exists a covering space of the figure eight with nontrivial abelian fundamental group?

4. Show that the 2-dimensional sphere with three points deleted cannot be a topological group.

5. Let M be a 4-dimensional closed simply connected manifold. Show that every continuous map $f : M \to M$ which is homotopic to the identity has a fixed point.

Answer the following with complete definitions or statements or short proofs.

6. State the Seifert-van Kampen Theorem.

7. State the Lefschetz Fixed Point Theorem.

8. State the Jordan Curve Theorem.

9. State the Arzela-Ascoli Theorem.

10. Let X be a finite complex. Find $\chi(X \times S^n)$.

11. Let $S_r(x, y)$ denote a circle of radius r centered at $(x, y) \in \mathbf{R}^2$. Let ΣR be the suspension over Hawaiian ring $R = \cup_{n=1}^{\infty} S_{\frac{1}{n}}(\frac{1}{n}, 0)$. Is ΣR

(a) locally path connected?

(b) locally simply connected?

(c) semi-locally simply connected?

12. State the Poincare Duality Theorem.

13. Define retraction and deformation retraction.

14. State the Universal Coefficient formula for homology.

15. Let X be a complete metric space such that no point is isolated. Can X be countable?