

**Topology Ph.D Exam**

May 2002

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper

1. State and prove the Contraction Mapping Theorem.
2. Show that  $\pi_1(S^2)$  is trivial.
3. Show that there is no retraction of the Mobius band onto its boundary.
4. Show that no covering of the 2-torus has the homotopy type of a figure eight.
5. Show that the space  $R^2 \setminus A$  is path connected for any countable set  $A$ .

Answer the following with complete definitions or statements or short proofs.

6. State the Baire Category Theorem.
7. State the Lefschetz Fixed Point Theorem.
8. State the Simplicial Approximation Theorem.
9. State the Arzela-Ascoli Theorem.
10. Let  $X$  be a finite complex. Find  $\chi(X \times S^3)$ .
11. State the Five Lemma.
12. State the Tietze Extension Theorem.
13. Define retraction and deformation retraction.
14. State the Mayer-Vietoris exact sequence for cohomology.
15. Let  $X$  be a complete metric space such that no point is isolated. Can  $X$  be countable?