Topology Ph.D. Exam
January 16, 2001

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Show that there is a continuous function from I onto I^2.

2. State and prove the Brouwer Fixed Point Theorem. You may assume that $H_n(S^n;Z) = Z$ for all $n \geq 1$.

3. Show that $H_n(S^n;Z) = Z$ and that $H_0(S^n;Z) = Z$ for $n \geq 1$.

4. Show that the fundamental group of S^n is trivial for $n > 1$.

5. Show that $\pi_1(P_n) = Z_2$ for $n > 1$, where P_n is n-dimensional real projective space.

Answer the following with complete definitions or statements or short proofs.

6. State the Seifert-van Kampen Theorem.

7. State the Hahn-Mazurkiewicz Theorem.

8. State the Lefschetz Fixed Point Theorem for compact simplicial complexes.

9. Define the *cone* of a space X. Define the *suspension* of X.

10. State the exact homology sequence for a pair of spaces.

11. State the Jordan Curve Theorem.

12. State the Contraction Mapping Theorem.

13. State the Ascoli (Arzela-Ascoli) Theorem.

14. Let A and B be disjoint closed sets in a metric space X. Show that there is a continuous function $f: X \to [0,1]$ such that $f^{-1}(1) = A$ and $f^{-1}(0) = B$.

15. Show that I and I^2 are not homeomorphic. Show that I^2 and I^3 are not homeomorphic.