Topology Ph.D. Exam
Sept 19, 2000

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. State and prove the Brouwer Fixed Point Theorem. You may assume some homology.

2. State and prove the Baire Category Theorem.

3. Give the Mayer-Vietoris sequence for homology. Use this to compute the homology groups for the n-sphere S^n.

4. Suppose that $f, g: X \to P$ are two maps and that P is a compact connected polyhedron. Show that there is an $\varepsilon > 0$ such that if $d(f(x), g(x)) < \varepsilon$ for all $x \in X$, then f and g are homotopic.

5. Show that $\pi_1(S^1) = Z$ where Z is the group of integers and S^1 is the circle.

Answer the following with complete definitions or statements or short proofs.

6. Define retraction and deformation retraction.

7. State Urysohn's Lemma. State the Tietze Extension Theorem.

8. State the Five Lemma.

9. Define the cone of a space X. Define the suspension of X.

10. State the exact homology sequence for a pair of spaces.

11. State the Jordan Curve Theorem.

12. Let A be a countable subset of R^2. Show that $R^2 \setminus A$ is arcwise connected.

13. Show that the Cantor set less its endpoints is homeomorphic to the irrationals in the unit interval. [Hint: Use the Cantor Ternary Function.]

14. Let V be a complete metric space such that no point is isolated. Can V be a countably infinite set?

15. Let $0 \to G \xrightarrow{g} H \xrightarrow{h} J \to 0$ be an exact sequence. What does this say about g and h? Prove your claims.