Topology Ph.D. Exam
May 16, 2000

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Show that there is no retraction of the Möbius band onto its boundary.

2. State and prove the Contraction Mapping Theorem.

3. Give the Mayer-Vietoris sequence for homology. Use this to compute the homology groups for the n-sphere S^n.

4. Suppose that $f, g: X \to S^n$ are two maps such that $f(x)$ and $g(x)$ are not antipodal for all $x \in X$. Show that f and g are homotopic.

5. Show that no covering space of the 2-torus has the homotopy type of a figure 8.

Answer the following with complete definitions or statements or short proofs.

6. Give a classification of the compact surfaces.

7. State Urysohn's Lemma. State the Tietze Extension Theorem.

8. State the Five Lemma.

9. Define the mapping cylinder of $f: X \to Y$. Define the mapping torus of $f: X \to X$.

10. State the exact homology sequence for a pair of spaces.

11. State the Jordan Curve Theorem.

12. Define retraction and deformation retraction.

13. State the Ascoli (Arzela-Ascoli) Theorem.

14. Let V be a complete metric space such that no point is isolated. Can V be a countably infinite set?

15. Let A be a countable subset of R^2. Show that $R^2 \setminus A$ is arcwise connected.