TOPOLOGY PRELIM EXAM. FALL 1993

Answer the following questions without giving proofs.

1. Let $X \subseteq Y$.
   a) What does it mean for X to be a retract of Y?
   b) What does it mean for X to be a deformation retract of Y?

2. Let K be a finite simplicial complex. Define $\chi(K)$, the Euler characteristic of K.

3. Define what it means for a topological space to be
   a) normal
   b) connected

4. State the Brouwer fixed point theorem.

Answer the following questions, giving proofs or counterexamples.

5. Let $p: X \to Y$ be a d-sheeted covering projection, where X and Y are finite simplicial complexes. Show that $\chi(X) = d \cdot \chi(Y)$.

6. Compute the fundamental group of a closed, orientable surface X of genus 3.

7. Let X be a closed, orientable surface of genus 3.
   a) Compute the homology groups of X.
   b) Compute the Euler characteristic of this surface X.

8. Show that every continuous map $f: S^2 \to S^1$ is homotopic to a constant map.

9. For $i = 1, 2, \ldots$ let P_i be a 2-dimensional plane in $\mathbf{R}^3$.
   a) Show that $S = \mathbf{R}^3 \setminus \cup_{i=1}^{\infty} P_i$ is not empty.
   b) Must S be an open set?

10. Let C be the Cantor set and let S be any compact metric space. Show that there is a continuous map $f: C \to S$ such that f is onto.