

TOPOLOGY PH.D. EXAM

May 18, 1993

PART I

State the following theorems:

- (1) Baire Category Theorem
- (2) Urysohn's Lemma
- (3) Brouwer Fixed Point Theorem
- (4) Jordan Separation Theorem for R^2
- (5) Schoenflies Theorem for R^2 . Does the corresponding statement hold for R^3 ? Explain.

PART II

For the following problems, show all work and write clearly. The proofs should be detailed, leaving no doubt about the correctness!

1. What is the universal cover of P^2 ? Prove your answer.
2. Prove: Let C be the Cantor set, and let X be any compact metric space. Then there exists a continuous, surjective function $f : C \rightarrow X$.
3. State and prove the Finite Simplicial Approximation Theorem.
4. Use the Seifert-van Kampen Theorem to compute the fundamental group of the Klein bottle.
5. (i) What is $\tilde{H}_0(X)$, when X is a two point space? (Do not prove.)
(ii) Use the Mayer-Vietoris sequence and (i) to compute the homology of S^n , for $n \geq 1$.