TOPOLOGY PRELIM EXAM, SUMMER, 1992

Answer the following questions without giving proofs.

1. Define what it means for a space X to be
   (a) connected
   (b) arcwise connected
   (c) simply connected

2. Let K be a finite simplicial complex. Define $\chi(K)$, the Euler characteristic of K.

3. Define the degree of a map $f : S^n \to S^n$. (You may assume that f is smooth.)

4. Let $K = K_1 \cup K_2$, where K is a simplicial complex and K_1, K_2, and $A = K_1 \cap K_2$ are subcomplexes of K. State the Mayer–Vietoris sequence (in either homology or cohomology) for (K, K_1, K_2, A).

Answering the following questions, giving proofs and examples.

5. Must a continuous map $f : S^4 \to S^4$ of non–negative degree have a fixed point?

6. Compute the fundamental group of a closed, non–orientable surfaces X of genus 2.

7. State and prove the Brouwer fixed point theorem.

8. Let U be an open subset of $\mathbf{R}^n$. Show that U is connected if and only if U is arcwise connected.

9. Show that no covering space fo the two–torus, $T^2 = S^1 \times S^1$, has the homotopy type of the figure–8, $S^1 \vee S^1$.

10. Compute the homology groups of the closed unit disk $D^n = \{x \in \mathbf{R}^n : ||x|| \leq 1\}$ and the unit sphere $S^n = \{u \in \mathbf{R}^{n+1} : ||u|| = 1\}$, for all $n \geq 0$. (Note that $D^0 = \{0\}$ and that $S^0 = \{-1, 1\}$.)