University of Florida

GNA QUALIFYING EXAM JANUARY 8, 2016

Name:

ID #:

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Directions: This is Part I of the PhD qualifying exam in Numerical Analysis or the semester Exam on MAD6407. If you are taking the PhD qualifying exam, you must take both Part I and Part II (numerical linear algebra) in one sitting. You must show all your work as neatly and clearly as possible and indicate the final answer clearly.

Problem	Possible	Points
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

(1) (20 points) A forth degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.

(2) (20 points) Consider the fixed point iteration

$$x_{n+1} = \phi(x_n), \qquad n - 0, 1, 2, \ldots$$

where

$$\phi(x) = Ax + Bx^2 + Cx^3$$

Given a positive number α, determine the constants A, B, C such that the iteration converges locally to $1/\alpha$ with order $p = 3$.

(3) (20 points) This problem has the following parts:

(a) Find α, β and γ so that the quadrature formula has a maximum degree of precision. What is the exact degree of precision of this quadrature formula?

$$\int_0^2 f(x)dx \approx \alpha(f(0) + f(2)) + \beta(f'(0) - f'(2)) + \gamma(f''(0) - f''(2)).$$

(b) Suggest your own quadrature formula for the integral

$$\int_0^2 f(x)\,dx.$$

Your formula should have expected degree of precision at least four (you don't have to compute to demonstrate that).

(4) (20 points) For the function

$$f(x) = \sqrt[m]{|x|}, \qquad m \neq \tfrac{1}{2}$$

in the interval $[-1, 1]$, find the polynomial of minimax approximation of degree 2. What is the minimax error? What happens if $m = \frac{1}{2}$?

(5) (20 points) Prove that Gaussian quadrature formula (for any n) in the interval $[-1, 1]$ and with weight $w(x) = 1$ is exact for all odd functions.