Logic Qualifing Exam
May 2013

Answer six questions and at least one from each section.

Section 1

1. Show that the theory of the class of infinite sets is complete and decidable, but is not finitely axiomatizable.

2. State and sketch a proof of the Los theorem about ultraproducts of models

3. Show that for any elementary chain $\{\mathcal{A}_i\}_{i\in\omega}$ of structures, $\mathcal{A}_i$ is an elementary submodel of the union $\mathcal{A}$.

Section 2

4. State and prove the Schroder-Bernstein Theorem

5. Show that Zorn's Lemma implies the Well-Ordering Principle.

6. Show that for any notion of forcing $\mathcal{P}$ and any countable set $\mathcal{D}$ of $\mathcal{P}$-dense sets, there exists a $\mathcal{D}$-generic $\mathcal{P}$-filter.

Section 3

7. Prove that if $X \subseteq \omega$ has an infinite computably enumerable subset A, then it has an infinite computable subset B.

8. Sketch the proof that there are $\leq_m$-incomparable c. e. sets.

9. Explain why the set WF of well-founded trees in ω^* and the set WO of well-orderings of ω are Π^1_1 complete and prove the Boundedness Principle, that any Σ^1_1 subset of WO is bounded below ω_1.