Logic Qualifing Exam
May 2010

Answer six questions and at least one from each section.

Section 1

1. Sketch the proof of Godel's Completeness Theorem for Predicate Logic and explain the use of Henkin constants (or Skolem functions).

2. Show that the theory of the class of infinite sets is complete and decidable, but is not finitely axiomatizable.

3. Sketch a proof of the Vaught test for categoricity using the Lowenheim-Skolem Theorem.

4. Show that if $\mathcal{A}$ and $\mathcal{B}$ are two countable atomic structures which are elementarily equivalent, then $\mathcal{A} \simeq_p \mathcal{B}$.

Section 2

5. (ZFC) For any infinite cardinal κ, $\kappa \times \kappa = \kappa$.

6. Describe the constructible universe L and sketch a proof that L satisfies the Continuum Hypothesis.

7. Show that for any set u and any countable set v, $FF(u, v)$ satisfies the countable (anti)-chain condition.

Section 3

8. Sketch a proof of Godel's incompleteness Theorem for Arithmetic. –You may assume the Representability Hypothesis.

9. Construct a simple r.e. set.

10. Show that the following are equivalent for any infinite $X \subseteq \omega$:
(a) X is recursively enumerable (that is, the range of a total recursive function;
(b) X is semirecursive (there is a recursive relation R such that, for all x, $x \in X \iff (\exists y) R(x, y)$.
(c) X is the domain of a partial recursive function.