

Logic exam

May 25, 2006

Please complete seven problems among the following, including one from each section.

1. General logic

- 1A. State the incompleteness theorem and explain the role of Goedel numbering in its proof.
- 1B. Sketch the proof of the completeness theorem for first order logic.
- 1C. Sketch the proof of Tarski's undefinability of truth.

2. Model theory

- 2A. Prove that the theory of dense linear orders without endpoints is complete.
- 2B. Find two models which are elementarily equivalent but not isomorphic.
- 2C. State and prove Tarski's criterion for elementary submodels.

3. Set theory

- 3A. Sketch the proof of L satisfying CH.
- 3B. Prove from Martin's Axiom that every set of reals of size aleph one has Lebesgue measure zero.
- 3C. Prove that every stationary subset of omega one can be split into aleph one many pairwise disjoint stationary subsets

4. Computability

- 4A. Show how to construct a simple recursively enumerable set A (meaning that $\omega - A$ has no infinite r.e. subset).
- 4B. Define the recursively enumerable set $K = \{a : \{a\}(a) \downarrow\}$. Use the S_1^1 function to show that K is many-one complete.
- 4C. Prove that the following are equivalent, for any nonempty $A \subset \omega$: (a) A is the domain of some partial recursive function ϕ . (b) A is the range of some (total) recursive function f .