

## Logic Qualifying Exam September 2005

Answer six questions and at least one from each section.

### Section 1

1. State and prove the compactness theorem for propositional logic.
2. Show that the theory of the class of infinite sets (in the language of equality) is complete and decidable, but not finitely axiomatizable.
3. State the Los Theorem for ultraproducts and sketch a proof.

### Section 2

4. Show that Zorn's Lemma implies that any set can be well-ordered.
5. Sketch a proof that  $ZFC \cup \{-CH\}$  is consistent.
6. Show that any countable union of countable sets is countable.

### Section 3

7. Sketch a proof of Godel's incompleteness Theorem for Arithmetic.
8. Show that an infinite set  $X \subseteq \omega$  is the domain of a partial recursive function if and only if  $X$  is the range of  $\phi$  for some partial recursive function  $\phi$ .
9. Show how to construct a simple recursively enumerable set  $A$  (meaning that  $\omega - A$  has no infinite r.e. subset)