Logic Qualifing Exam
September 2004

Answer 7 questions.

1. Sketch a proof that every truth function $E : \{T, F\}^n \to \{T, F\}$ can be represented by a propositional sentence.

2. State the (Countable) Compactness for First Order Logic and sketch the proof using Henkin completeness.

3. Sketch a proof that the theory of dense linear orderings without end points satisfies Quantifier Elimination and hence is complete and decidable.

4. State and prove the Omitting Types Theorem.

5. Show that Zorn's Lemma implies the Axiom of Choice.

6. Show that for any notion of forcing P and any countable set D of P-dense sets, there exists a D-generic P-filter.

7. Define the constructible universe L and sketch a proof that L satisfies the Continuum Hypothesis.

8. Sketch a proof of the Undecidability of (Peano) Arithmetic.

9. Show that an infinite set $X \subseteq \omega$ is the domain of a partial recursive function if and only if X is semirecursive (that is, $x \in X \iff (\exists y)R(x, y)$ where R is a recursive relation).

10. Show how to construct a simple recursively enumerable set which is neither recursive nor m-complete.