

Logic Qualifying Exam

September 2000

Answer 7 questions, including at least one from each of the four segments.

1. General Logic

1. State Gödel's incompleteness theorem, and briefly sketch its proof.
2. Show that any complete, finitely axiomatizable first-order theory is decidable.
3. Give an example of two countable models which are elementarily equivalent, but not isomorphic.

2. Model Theory

1. State and sketch a proof of the completeness theorem.
2. State and sketch a proof of the Łos theorem about ultraproducts of models.

3. Set Theory

1. Prove that a countable union of countable sets is countable.
2. Sketch the proof that $L \models \text{GCH}$.
3. Prove that if κ is a regular, uncountable cardinal then any diagonal intersection of closed and unbounded subsets of κ is closed and unbounded.

4. Recursion Theory

1. Show that any total function $f: \omega \rightarrow \omega$ with a recursively enumerable graph is recursive. Show that this is not true for partial functions.
2. Explain why, for any $A \subset \omega$, there are at most countably many sets B such that $B <_T A$.
3. State and prove Kleene's Recursion (or Fixed Point) Theorem.