Logic Qualifing Exam
May 25, 2000

Answer 7 questions, including at least one from each of the four segments.

1. General Logic

1. State Gödel's incompleteness theorem, and explain the role of Gödel numbering in its proof.
2. Prove that there is no finite set Γ of sentences in the language of groups such that a group G satisfies all the sentences in Γ if and only if G is not torsion free. [The group G is torsion free if $g^n \neq e$ for all $g \neq e$ in G.]
3. Show that any complete, finitely axiomatizable first-order theory is decidable.
4. Give an example of two models which are elementarily equivalent, but not isomorphic.

2. Model Theory

1. State and sketch a proof of the downward Lowenheim-Skolem theorem.
2. State the omitting types theorem, and use it to prove that any ω-complete theory fo arithmetic has an ω-model.
3. State and sketch a proof of the Los theorem about ultraproducts of models.

3. Set Theory

1. Prove that a countable union of countable sets is countable.
2. Sketch the proof that $L \models \mathrm{GCH}$.
3. Prove that if κ is a regular, uncountable cardinal then any diagonal intersection of closed and unbounded subsets of κ is closed and unbounded.

4. Recursion Theory

1. Show that a function $h\colon \omega \to \omega$ is (partial) recursive if and only its graph is recursively enumerable; and that if h is total then its graph is recursive.
2. State the halting problem, and prove that it is undecidable.
3. Explain why, for any $A \subset \omega$, there at at most countably many sets B such that $B <_{\mathrm{T}} A$.