

Logic Qualifying Exam
May 27, 1993 (???)

This examination is divided into 4 segments, with 3 questions in each segment. Answer any 2 questions from each segment.

1. General Logic

1. State Gödel's incompleteness theorem. Explain the importance of Gödel numbering.
2. State and prove the compactness theorem for Predicate logic. You may assume the completeness theorem.
3. Prove that if T is a countable theory with arbitrarily large finite models then T has an infinite model.

2. Model Theory

1. State and sketch the proof of the Los theorem on ultraproducts.
2. State and sketch a proof of the elementary substructure version of the downward Lowenheim-Skolem Theorem.
3. Prove that the theory of dense linear orders without endpoints is complete.

3. Set Theory

1. Prove that the well ordering principle implies the axiom of choice.
2. Prove, using the axioms of ZFC, that if X is any set then the transitive closure of X is also a set.
3. Show that a countable union of countable sets is countable, pointing out any use of the axiom of choice.

4. Recursion Theory

1. Show that a function $\phi : \omega \rightarrow \omega$ is (partial) recursive iff its graph is recursively enumerable, and that if ϕ is total and recursive then its graph is recursive.
2. State the halting problem, and show that it is undecidable. You may use the fact that there is a recursive enumeration of the (partial) recursive functions.
3. Show that if $A \neq \emptyset$ is the domain of a partial recursive function f then there is a total recursive function g such that A is the range of g .