Logic Qualifying Exam
September 10, 1992

This examination is divided into 4 segments, with 3 questions in each segment. Answer any 2 questions from each segment.

1. General Logic

1. Prove that there is no single sentence ϕ in the language of ring theory such that a ring R is a model of ϕ if and only if R has characteristic 0.

2. State and prove the compactness theorem for Predicate logic. You may assume the completeness theorem.

3. State and sketch a proof of Gödel's incompleteness theorem.

2. Model Theory

1. Show that any two elementarily equivalent, countably saturated models are isomorphic.

2. State and sketch a proof of the downward Lowenheim-Skolem Theorem.

3. State the Omitting types theorem, and use it to show that any ω-complete theory of arithmetic has an ω-model.

3. Set Theory

1. Show that a countable union of countable sets is countable. Explain any use of the axiom of choice.

2. Show that $2^\kappa = \kappa^\kappa$ for all infinite κ.

3. Use Fodor's theorem to prove that the diagonal intersection of a sequence of closed, unbounded sets is closed and unbounded.

4. Recursion Theory

1. Show that a function $\phi : \omega \to \omega$ is (partial) recursive iff its graph is recursively enumerable, and that if ϕ is total and recursive then its graph is recursive.

2. Prove that there is a subset of ω which is recursively enumerable but not recursive.

3. State the normal form theorem for partial recursive functions and use it to prove the enumeration theorem.