Logic Qualifier Fall 1989

DO 2 FROM EACH SECTION

I. GENERAL LOGIC

1. State and prove the Löwenheim-Skolem Theorem for countable languages.
2. State and prove Herbrand's Theorem.
3. Suppose that $\mathcal{N}$ is a submodel of $\mathcal{M}$. Prove that if A is a quantifier-free sentence, then

$$\mathcal{N} \models A \quad \Longleftrightarrow \quad \mathcal{M} \models A$$

II. SET THEORY

4. Prove the Mostowski Collapsing Lemma: If (M, E) is a well-founded model of extensionality, then there is a transitive set X and an isomorphism $\phi : (M, E) \cong (X, \in)$
5. Sketch a proof of $Con(ZFC) \implies Con(ZFC + \neg CH + \omega_1 = \omega_1^L)$. You may assume general theorems about forcing, eg. the Truth Lemma.
6. Define the constructible universe L and prove that for all α, L_α is transitive.

III. RECURSION THEORY

A infinite

7. Prove that A is recursive if and only if there is a strictly increasing onto $f : \mathbb{N} \to A$.
8. Prove that there exist recursively enumerable, recursively inseparable sets.
9. Prove that there is no primitive recursive function $F(e, n)$ such that for every primitive recursive function $f(n)$ there is an e such that $F(e, n) = f(n)$ for all n.