MATHEMATICAL LOGIC PH.D. EXAM

Fall 1978

Do any 8 problems.

1. Use truth tables to demonstrate the Cut Rule: $\{P \vee Q, \neg P \vee R \models Q \vee R\}$.

2. State Godel's Completeness Theorem for Propositional Logic and sketch the proof for a countable theory. (You may assume the Deduction Lemma.)

3. State and prove the Compactness Theorem for Predicate Logic. (You may assume the Completeness Theorem.)

4. Explain why the Theory of Dense Linear Orderings without Endpoints is complete. State the main results needed for the proof.

5. Prove that every primitive recursive function is representable in formal arithmetic.

6. Define the ultraproduct $\Pi_{i \in I} M_i / U$ and state Los' Theorem.

7. Explain why there is no Enumeration Theorem for the Primitive Recursive Functions.

8. State the Well-Ordering Principle and the Axiom of Choice and prove that the Well-Ordering Principle implies the Axiom of Choice.

9. Prove that $\omega^{<\omega}$ is countable, where $\omega^{<\omega}$ is the set of finite strings of natural numbers.

10. Suppose $2 \leq \kappa \leq \lambda$ and λ is infinite. Show that $2^\lambda = \kappa^\lambda$.