Functional Analysis PhD Exam Jan 2011

<u>Do only 5 of the 8 problems</u>

① State the Krein-Milman Theorem and sketch the proof.
Correct

② Prove that if A is an element of a Banach algebra $\mathfrak{A}$ then the spectrum of A is a non-empty closed subset of the closed disk in $\mathbb{C}$ with center 0 and radius $\|A\|$.

③ State the "Function Representation Theorem" involving an abelian C^*-algebra $\mathfrak{A}$ in terms of $C(\mathcal{P}(\mathfrak{A}))$, where $\mathcal{P}(\mathfrak{A})$ is the set of all pure states of $\mathfrak{A}$. Sketch the proof of this theorem.

④ State and sketch the proof of the theorem involving $\mathcal{A} \cong C(X)$, where $\mathcal{A}$ is an abelian von Neumann algebra and X is an extremely disconnected compact Hausdorff space. Forgot to Mention Borel maps

⑤ Give the main elements of the Gelfand-Neumark-Segal construction. State the theorem involving a state ρ of a C^*-algebra $\mathfrak{A}$ and a cyclic representation π_ρ of $\mathfrak{A}$ on a Hilbert space $\mathcal{H}_\rho$. List the main steps in the proof. Correct

⑥ Let $\mathfrak{A}$ be a C^*-algebra. State and prove equivalent conditions for $A \in \mathfrak{A}^+$. correct

⑦ Let H be a self-adjoint operator on a Hilbert space. Define the Cayley transform $\mathfrak{U}(H)$ and show that $H \to \mathfrak{U}(H)$ on the self-adjoint operators is strong-operator continuous.

⑧ Let $\mathcal{A}$ be the algebra of multiplications by elements of $L_\infty(S, \mathcal{S}, m)$ on $L_2 = L_2(S, \mathcal{S}, m)$. If T is a bounded operator on L_2 commuting with $\mathcal{A}$, show $T \in \mathcal{A}$. Assume $m(S) < \infty$. correct