Name: Keith Copenhaver

1. Let $f(n)$ be the number of permutations of length n that contain no cycles of length exactly two.

   (a) Find the exponential generating function of the sequence $f(n)$. You can assume that $f(0) = 1$.

   (b) Find $\lim_{n \to \infty} f(n)/n!$.

2. We select an odd number of people from a group on n people to form a committee. Then we select an even number of people from this committee to serve on a subcommittee. (Zero is an even number.) In how many ways can we do this?

3. Let T be a tree in which no path is longer than k. Is it true that there is a vertex $v \in T$ that is contained in all paths of length k in T?

4. Let $X_n(S)$ be the size of the subset S of $[n]$ selected uniformly, that is, each element of n has fifty percent chance to be selected. Compute $VAR(X_n)$.

5. Let f_n be the number of all parts of all compositions of n. So $f_1 = 1$, $f_2 = 3$, and $f_3 = 8$. Find and prove an explicit formula for f_n.

6. Let m_n be the number of ways to have a group of n people split into nonempty subsets, to have each subset sit down around a circular table, then to arrange the tables in a circle. Find the exponential growth rate of the sequence $m_n/n!$. Two arrangements are considered identical if each person has the same left neighbor in them, and each table has the same left neighbor in them.

7. Prove that every convex polyhedron has two faces that have the same number of vertices.

8. Let p be a permutation that can be decomposed into the disjoint union of k increasing subsequences, but not into the disjoint union of $k-1$ or fewer increasing subsequences. What can be said about the length of the longest decreasing subsequence of p?