1. Prove that there exists a simple k-regular graph on n vertices if and only if nk is even, and $n \geq k+1$.

2. Find a closed formula (no summation signs) for the generating function $\sum_{n\geq 0} g(n)x^n/n!$, where $g(n)$ is the number of permutations of length n with all cycle lengths even.

3. Show that a planar graph in which every face has an even number of vertices must be bipartite.

4. Let $f(n)$ be the number of simple graphs on vertex set $\{1, 2, \cdots, n\}$ in which each connected component is a cycle. Find a closed form for the generating function $\sum_{n\geq 0} f(n)x^n/n!$.

5. How many Hamiltonian cycles does the graph $K_{n,n}$ have?

6. Let $h(n)$ be the number of compositions of the integer n into odd parts. Find the generating function $H(x) = \sum_{n\geq 1} h(n)x^n$.

7. Find a formula for the number of partitions of the set $\{1, 2, \cdots, n\}$ in which each block contains more than one element. Your answer can contain the symbol $B(i)$ for the number of all partitions of $[i]$.

8. Let $d \geq 2$ be an integer, and let P be the poset of all vectors of length d with non-negative integer coordinates in which $\mathbf{x} \leq \mathbf{y}$ if $x_i \leq y_i$ for all i.

   (a) Let n be any positive integer. Does P contain an antichain of n elements?

   (b) Does P contain an infinite antichain?