Ph.D. Exam in Analysis   May 2012

Be sure to carefully present all work in a neat and logical fashion. Do not leave any gaps. State clearly theorems used in your proofs. Print your name on each sheet turned in.

(1) Let μ be a real-valued signed measure on a σ-algebra Σ. Does there exist a set in Σ on which μ attains its minimum value? Prove or disprove.

(2) Let X be a Banach space. Show that there exists a compact T_2 space S such that X is isometrically isomorphic to a closed linear subspsce of $C(S)$. [Hint: Examine the unit ball of the dual space X^* with the appropriate topology and the action of X on X^*]

(3) Let $\{f_n\}$ be a sequence of integrable functions on S such that $\sum \int |f_n|\, d\mu < \infty$. Give all details concerning the convergence of $\sum f_n(s)$, $s \in S$.

(4) State and prove the Fubini theorem.

(5) Let π be the natural map from the Banach space X into its second dual X^{**}. Show $\pi(X)$ is closed in the norm topology of X^{**}.

(6) Show that the continuous functions on $[0,1]$ are dense in $L^1[0,1]$ (Lebesgue measure).
[Hint: First show this is true for the indicator function of a Lebesgue measurable subset of $[0,1]$]