Logic Qualifying Exam, May 2017

Answer six questions, at least one in each section.

Section 1

1. Provide a complete description of a formal proof system for first order logic.

2. Sketch the proof of the completeness theorem for first order logic.

3. State and prove Gödel's diagonal lemma.

Section 2

1. Sketch the proof that the Continuum Hypothesis holds in the constructible universe L.

2. Show that Zorn's lemma implies the well-ordering principle.

3. Sketch the proof of consistency of the Continuum Hypothesis using forcing.

Section 3

1. Show that every nonempty Σ_1-definable set of natural numbers is the range of a total recursive function.

2. Sketch the proof of Friedberg–Muchnik theorem: there are two subsets of ω which are incomparable in the sense of the Turing ordering.

3. Explain what a simple set is and provide an example with a proof.