

## Logic PhD Exam, May 2023.

Solve 5 problems of the following; at least one from each section.

### A. Set Theory.

1. State Martin's Axiom for  $\aleph_1$  and prove that it implies the negation of the Continuum Hypothesis.
2. Give the definition of a standard Borel space. Sketch the proof that any two uncountable standard Borel spaces are Borel isomorphic.
3. Let  $\kappa$  be an uncountable cardinal. Show that there is an ultrafilter on  $\kappa$  containing no sets of cardinality smaller than  $\kappa$ .

### B. Computability.

1. State two different common definitions of the class of computable functions and sketch the proof that they give the same class.
2. Define the many-one reducibility and sketch the proof that there is a many-one degree strictly between  $\mathbf{0}$  and  $\mathbf{0}'$ .
3. Prove that every infinite computably enumerable subset of the natural numbers contains an infinite computable subset.

### C. Model theory.

1. What is quantifier elimination for theories? Give an example of a theory which has it, and prove that it has it. Give an example of theory which does not have it, and prove that it does not have it.
2. State the Łoś' theorem and prove it.
3. Give an example of a relational Fraisse class which has the disjoint amalgamation property, and an example of a relational Fraisse class which does not have it.