

Logic PhD Exam, January 2022.

Solve 5 problems of the following; at least one from each section.

A. Set Theory.

1. Sketch the proof that the constructible universe satisfies the Continuum Hypothesis.
2. State Martin's Axiom for a cardinal κ . Show that it proves that the continuum has cardinality larger than κ .
3. Given a Polish space X , explain how the hyperspace $K(X)$ of all compact subsets of X is topologized. Sketch the proof that Vietoris topology is Polish.

B. Computability.

1. What are Turing degrees? Sketch the proof that for a Turing degree t , the jump t' is of strictly larger Turing degree.
2. Provide at least two conceptually different definitions of the class of computable functions and show that they are equivalent.
3. What is Post's problem? Provide the answer to the Post's problem and an outline of the proof.

C. Model theory.

1. What is a Fraissé class? State the Fraissé theorem regarding the existence of a limit model for such a class.
2. What is quantifier elimination? Provide an example of a theory which has infinite models and quantifier elimination. Sketch a proof that the theory has quantifier elimination.
3. Find a complete theory with more than one countable model up to isomorphism. Provide two nonisomorphic countable models of the theory.