

**Logic PhD Exam, August 2019.**

Solve 5 problems of the following; at least one from each section.

**A. Set Theory.**

1. Sketch the construction of any model of ZFC and the negation of the Continuum Hypothesis.
2. Define the class of  $\Delta_0$  formulas. Prove carefully that if  $\phi(x)$  is a  $\Delta_0$  formula with one free variable and  $M$  is any transitive set and  $x \in M$ , then  $\phi(x)$  is equivalent to  $M \models \phi(x)$ .
3. Prove that if  $X$  is a Polish space and  $A_0, A_1 \subset X$  are disjoint analytic sets, then there are disjoint Borel sets  $B_0, B_1 \subset X$  such that  $A_0 \subset B_0$  and  $A_1 \subset B_1$ .

**B. Computability.**

1. Define  $\mathbf{0}'$  (zero jump) and show that it is a computably enumerable set which is not computable.
2. Define the many-one reducibility and sketch the proof that there is a many-one degree strictly between  $\mathbf{0}$  and  $\mathbf{0}'$ .
3. What is Goedel's diagonalization lemma? Prove the lemma.

**C. Model theory.**

1. State the downward Loewenheim–Skolem theorem and prove it.
2. State the Los' theorem and prove it.
3. Find a complete theory with exactly one infinite countable model up to isomorphism. Prove this property of the theory.