Logic PhD Exam, May 2019.

Solve 5 problems of the following; at least one from each section.

A. Set Theory.

1. Sketch the construction of any model of ZFC and the Continuum Hypothesis.

2. Define the class of Δ_0 formulas. Prove carefully that if $\phi(x)$ is a Δ_0 formula with one free variable and M is any transitive set and $x \in M$ then $\phi(x)$ is equivalent to $M \models \phi(x)$.

3. Provide an example of an analytic set which is not coanalytic. Provide a proof of these properties of the set.

B. Computability.

1. Provide an example of a computably enumerable set which is not computable. Provide a proof of these properties of the set.

2. What is Goedel's diagonalization lemma? Prove the lemma.

3. What is Post's problem? Provide the answer to the Post's problem and an outline of the proof.

C. Model theory.

1. State the downward Loewenheim–Skolem theorem and prove it.

2. State the Los' theorem and prove it.

3. Find a complete theory with more than one countable model up to isomorphism. Provide two nonisomorphic countable models of the theory.