Logic Qualifying Exam, May 2017

Answer six questions, at least one in each section.

Section 1

1. State and prove Tarski's criterion for elementarity of submodels.
2. State and prove the downward Loewenheim–Skolem theorem.
3. Prove that the models $\langle \mathbb{Z}, \leq \rangle$(integers with the usual ordering) and $\langle \mathbb{Z} + \mathbb{Z}, \leq \rangle$ (two copies of integers with the usual ordering, one following the other) are elementarily equivalent.

Section 2

1. Prove that for any cardinal κ, the cofinality $cf(2^\kappa) > \kappa$.
2. Show that the wellordering principle implies the Zorn's lemma.
3. Show that a c.c.c. forcing preserves all cardinals.

Section 3

1. Show that every computably enumerable set has an infinite computable subset.
2. Show that there are two subsets of ω which are incomparable in the sense of the Turing ordering.
3. Explain what a Π^1_1 complete set is and provide an example with a proof.